Cascades in Networks and Agent Based Modeling

I. Makarov & L.E. Zhukov

BigData Academy MADE from Mail.ru Group

Network Science

Lecture outline

1. Diffusion of innovation
2. Influence propagation models
   - Independent cascade model
   - Linear threshold model
3. Influence maximization problem
   - Submodular function optimization
4. Spatial Segregation

Diffusion process

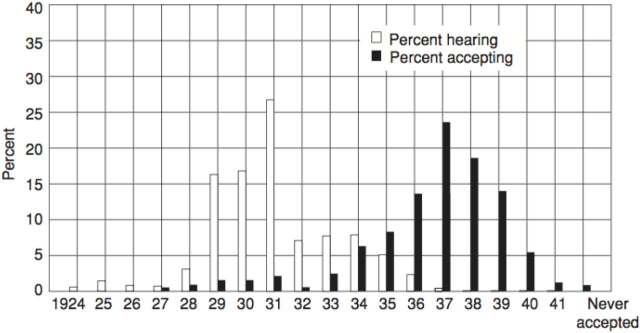
Propagation process:

- Viral propagation:
  - virus and infection
  - rumors, news
  - information
- Threshold (agent decision) models:
  - adoption of innovation
  - joining politcal protest
  - purchase decision
  - cascading failures

Local individual decision rules will lead to very different global results.
"microscopic" changes $\rightarrow$ "macroscopic" results

Diffusion of innovation

Ryan-Gross study of hybrid seed corn delayed adoption - diffusion of innovation

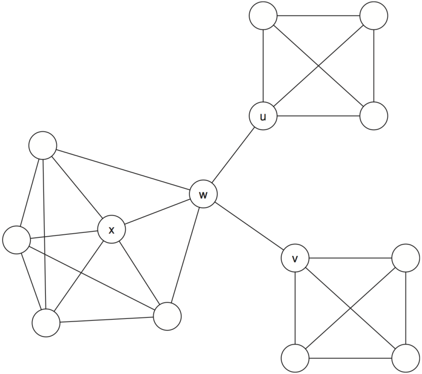


Information effect vs adopting of innnovation

Ryan and Gross, 1943

Diffusion of innovaton

Information (awareness) vs adoption (decision) spreading



Network coordination game

Local interaction game: Let u and v are players, and A and b are possible strategies

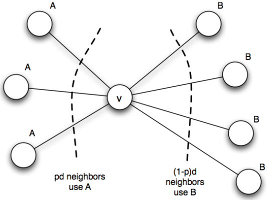
Payoffs

- if u and v both adopt behavior A, each get payoff $a > 0$
- if u and v both adopt behavior B, each get payoff $b > 0$
- if u and v adopt opposite behavior, each get payoff 0

$$\begin{array}{cc|c|c|} & & \multicolumn{2}{c}{w} \\ & & A & B \\ \cline{3-4} \multirow{2}{*}{v} & A & a, a & 0, 0 \\ \cline{3-4} & B & 0, 0 & b, b \\ \cline{3-4} \end{array}$$

Threshold model

Network coordination game, direct-benefit effect



v \ w	A	B
A	a, a	$0, 0$
B	$0, 0$	b, b

Node v to make decision A or B, p - portion of type A neighbors to accept A:

$$a \cdot p \cdot d > b \cdot (1 - p) \cdot d$$

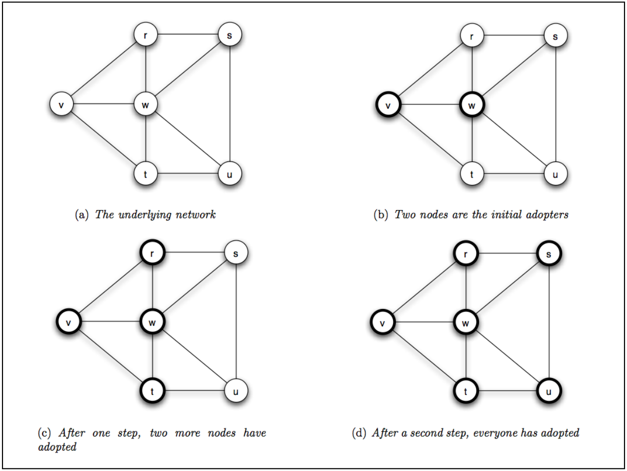
$$p \geq b/(a + b)$$

Threshold:

$$q = \frac{b}{a + b}$$

Accept new behavior A when $p \geq q$

Cascade - sequence of changes of behavior, "chain reaction"



(a) *The underlying network*

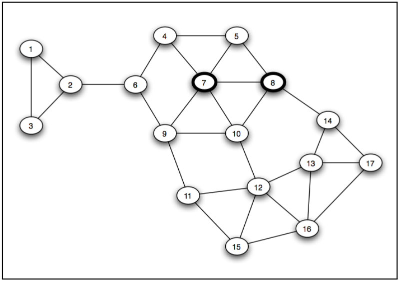
(b) *Two nodes are the initial adopters*

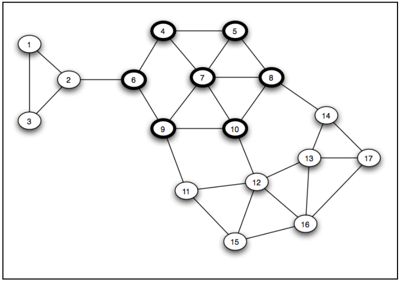
(c) *After one step, two more nodes have adopted*

(d) *After a second step, everyone has adopted*

Let $a = 3, b = 2$, threshold $q = 2/(2 + 3) = 2/5$

Cascade propagation



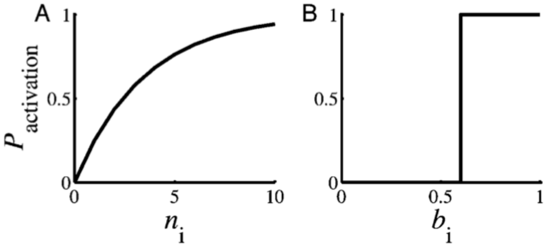


- Let $a = 3$, $b = 2$, threshold $q = 2/(2+3) = 2/5$
- Start from nodes 7,8: $1/3 < 2/5 < 1/2 < 2/3$
- Cascade size - number of nodes that changed the behavior
- Complete cascade when every node changes the behavior

Influence response

Two models:

- Independent Cascade Model (diminishing returns)
- Linear Threshold Model (critical mass)



$$P(n) = 1 - (1 - p)^n \qquad P(b) = \delta(b > b_0)$$

Influence response: diminishing returns and threshold

D. Kempe, J. Kleinberg, E. Tardos, 2003

Independent cascade model

- Initial set of active nodes S_0
- Discrete time steps
- On every step an active node v can activate connected neighbor w with a probability $p_{v,w}$ (single chance)
- If v succeeds, w becomes active on the next time step
- Process runs until no more activations possible

If $p_{v,w} = p$ it is a particular type of SIR model, a node stays infected for only one step

D. Kempe, J. Kleinberg, E. Tardos, 2003

Linear threshold model

- Influence comes only from NN $N(i)$ nodes, w_{ij} influence $i \to j$
- Require $\sum_{j \in N(i)} w_{ji} \leq 1$
- Each node has a random acceptance threshold from $\theta_i \in [0, 1]$
- Activation: fraction of active nodes exceeds threshold

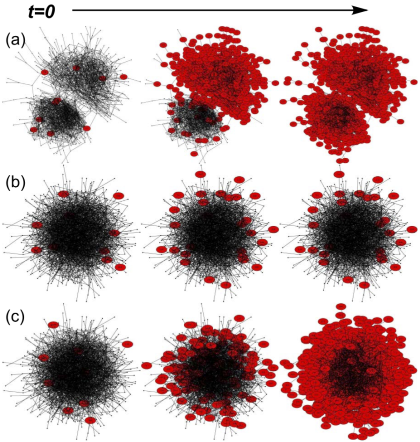
$$\sum_{active\ \ j \in N(i)} w_{ji} > \theta_i$$

- Initial set of active nodes A_o, iterative process with discrete time steps
- Progressive process, only nonactive $\to$ *active*

D. Kempe, J. Kleinberg, E. Tardos, 2003

Cascades in random networks

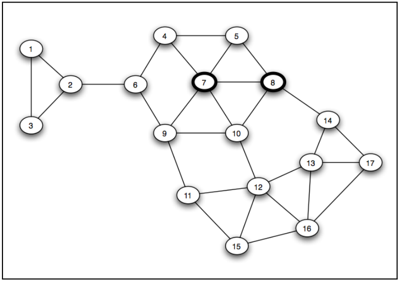
multiple seed nodes

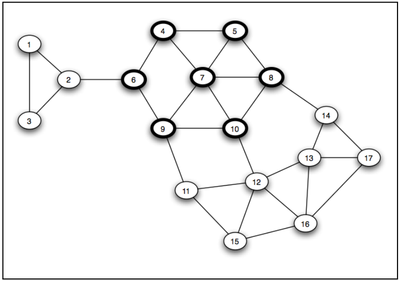


(a) Empirical network; (b), (c) - randomized network

P. Singh, 2013

Influence maximization problem





- Initial set of active nodes A_o
- Cascade size $\sigma(A_o)$ - expected number of active nodes when propagation stops
- Find k-set of nodes A_o that produces maximal cascade $\sigma(A_o)$
- k-set of "maximum influence" nodes
- NP-hard

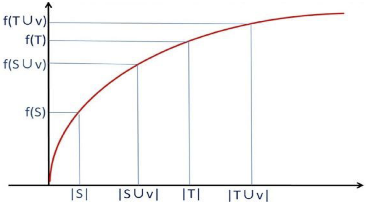
D. Kempe, J. Kleinberg, E. Tardos, 2003, 2005

Submodular functions

- Set function f is submodular, if for sets S, T and $S \subseteq T$, $\forall v \notin T$

$$f(S \cup \{v\}) - f(S) \geq f(T \cup \{v\}) - f(T)$$

- Function of diminishing returns ("concave property")
- Function f is monotone, $f(S \cup \{v\}) \geq f(S)$



Submodular functions

Theorem

Let F be a monotone submodular function and
let S^* be the k-element set achieving maximal f.
Let S be a k-element set obtained by repeatedly, for k-iterations, including an element producing the largest marginal increase in f.

$$f(S) \geq (1 - \frac{1}{e}) f(S^*)$$

Nemhauser, Wolsey, and Fisher, 1978

Influence maximization

- Cascade size $\sigma(S)$ is submodular function (D. Kempe, J. Kleinberg, E. Tardos, 1993)

$$\sigma(S) \geq (1 - \frac{1}{e})\sigma(S^*)$$

- Greedy algorithm for maximum influence set finds a set S such that its influence set $\sigma(S)$ is within $1/e = 0.367$ from the optimal (maximal) set $\sigma(S^*)$, $\sigma(S) \geq 0.629\sigma(S^*)$

Influence maximization

Approximation algorithm

Algorithm: Greedy optimization

Input: Graph $G(V, E)$, k

Output: Maxumum influence set S

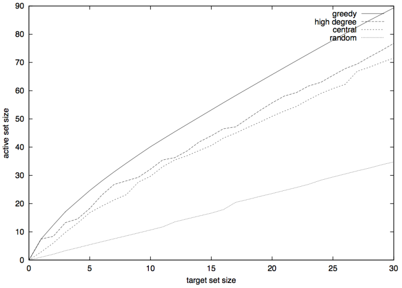
Set $S \leftarrow 0$

for $i = 1 : k$ **do**

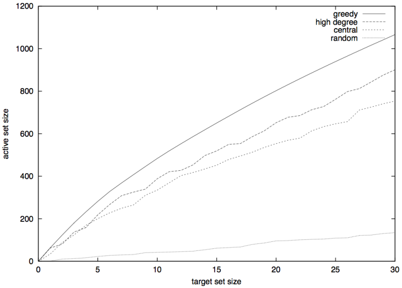
  select $v = \arg\max_{u \in V \setminus S} (\sigma(S \cup \{u\}) - \sigma(S))$

  $S \leftarrow S \cup \{v\}$

Experimental results



Independent cascade model

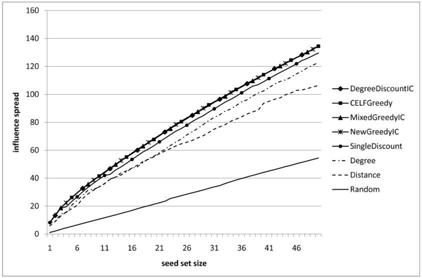


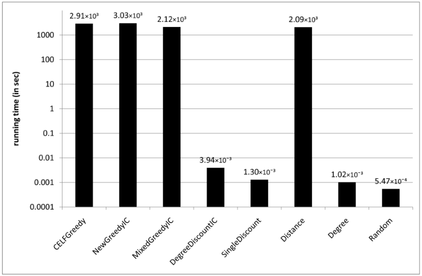
Linear threshold model

network: collaboration graph 10,000 nodes, 53,000 edges

D. Kempe, J. Kleinberg, E. Tardos, 2003

Computational considerations





Independent cascade model: influence spread and running time

W. Chen et.al, 2009

Lecture outline

1. Diffusion of innovation
2. Influence propagation models
   - Independent cascade model
   - Linear threshold model
3. Influence maximization problem
   - Submodular function optimization
4. Spatial Segregation

Agent Based Modeling (ABM)

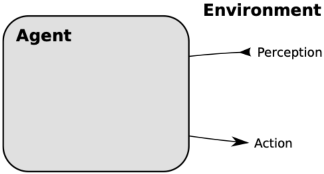
An agent-based model (ABM) is a class of computational models for simulating the actions and interactions of autonomous agents

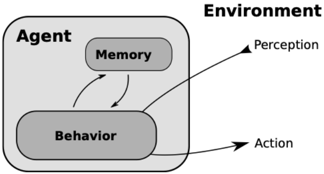
- Agent-based models consist of dynamically interacting rule-based agents.
- Simple agent behaviors (rules) generate complex system behaviors
- Real world system becoming very complex and interdependent
- Decentralization of decision making, deregulations.
- Available data and computational power for micro simulations

What is an agent?

Agents are autonomous decision-making units with diverse charactristics

- A discrete entity with its own goals and behavior
- Autonomous, capable to adapt and modify behavioir
- Decisions made independently by each engine
- Agents can be homogeneous or diverse and heterogeneous
- Can have memory and internal models
- Examples: people, groups, organizations, systems of robots etc





Agent based simulations

- Agent based model consists of a set of user defined agents, a set of agent relationships and environment
- No central controller or authority exists for the system
- Independent move and interaction by any agent
- Local interaction among agents
- Various topologies connect agents with their neighbors (fee space, grid, network, GIS)
- Optimization can be done for the system globally

When use agent based modeling?

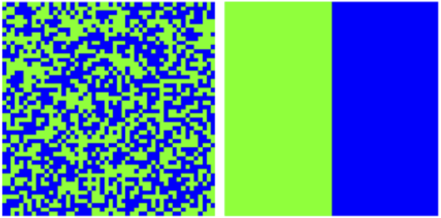
- When there is a natural representation as agents
  - When there are decision and behaviors that can be defined discretely (with boundaries)
  - When it is important that agents adapt and change their behavior
  - When it is important that agents learn and engage in dynamic strategic behavior
  - When it is important that agents have a dynamic relationships with other agents, and agent relationships form and dissolve
  - When it is important that agents have a spatial component to their behaviors and interactions
- When the past is no predictor of the future
- When scale-up to arbitrary levels is important
- When process structural change needs to be a result of the model, rather than an input to the model

Spatial model of segregation

"Dynamic Models of Segregation", Thomas Schelling, 1971

- Micro-motives and macro-behavior
- Personal preferences lead to collective actions
- Global patterns of spatial segregation from homophily at a local level
- Segregated race, ethnicity, native language, income
- Cities are strongly racially segregated. Are people that racists?
- Agent based modeling: agents, rules (dynamics), aggregation

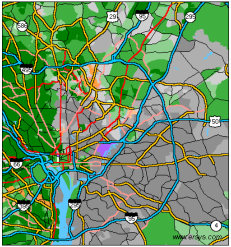
Segregation



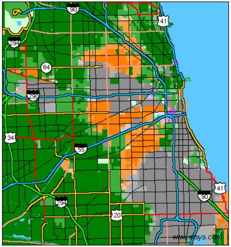
Integrated pattern   Segregated pattern

Racial segregation

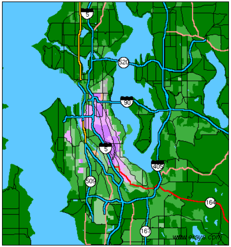
New York



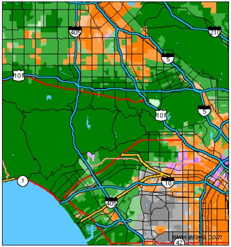
Washington



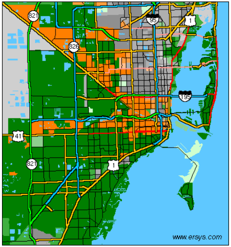
Chicago



Seattle

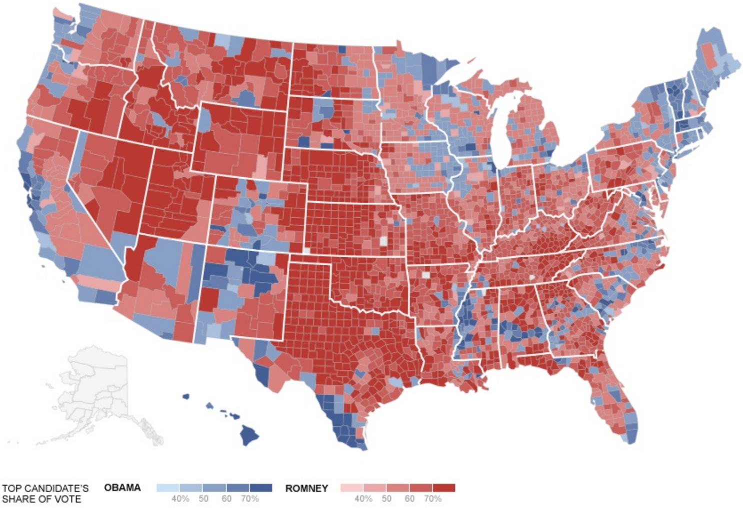


Los Angeles



Miami

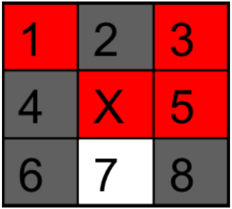
2012 US Presidential Elections Map



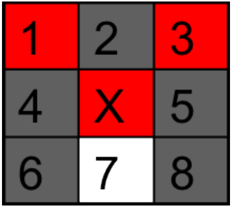
Schelling's model of segregation

- Population consists of 2 types of agents
- Agent reside in the cells of the grid (2-dimensional geography of a city), 8 neighbors
- Some cells contain agents, some unpopulated
- Every agent wants to have at least some fraction of agents (threshold) of his type as neighbor (satisfied agent)
- On every round every unsatisfied agent moves to a satisfactory empty cell.
- Continues until everyone is satisfied or can't move

Spatial segregation



satisfied agent



unsatisfied agent

- preference threshold $\lambda = 3/7$

Model

- N - nodes, θ - fraction of occupied by A and B

$$n_A + n_B = \theta \cdot N$$

- Proportion of "unlike" nearest neighbors, $k_i = \#NN$

$$P_i = \begin{cases} \#n_B/k_i, \text{if} \;\; i \in A \\ \#n_A/k_i, \text{if} \;\; i \in B \end{cases}$$

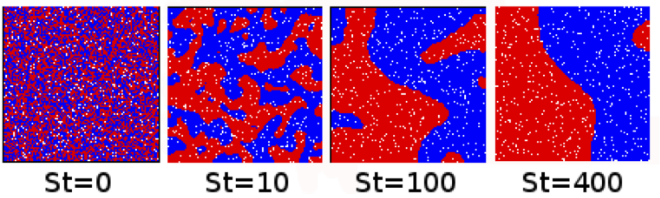
- Utility function, λ - sensitivity (tolerance threshold) level

$$u_i = \begin{cases} 1, \text{if} \;\; P_i \leq \lambda \\ 0, \text{if} \;\; P_i > \lambda \end{cases}$$

- Every node moves to maximize its utility

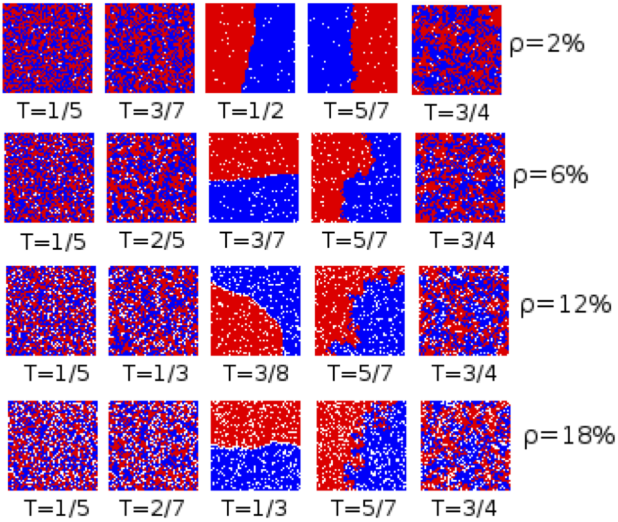
Spatial segregation

vacancy 5%, tolerance $\lambda = 0.5$



L. Gauvin et.al. 2009

Spatial segregation



L. Gauvin et.al. 2009

Algorithm

- time steps $1..T$
- At every time step randomly select an agent, compute utility
- If utility is $u = 0$ move to an empty location to maximize utility
- Movements: 1) random location 2) nearest available location
- Repeat until either all utilities are maximized $\sum_i u_i = \theta N$ or reaches "frozen" state, no place to move, then $\sum_i u_i < \theta N$
- Total utility of society

$$U = \sum_i u_i$$

Measuring segregation

- Schilling's solid mixing index

$$M = \frac{1}{n_A + n_B} \sum_i P_i$$

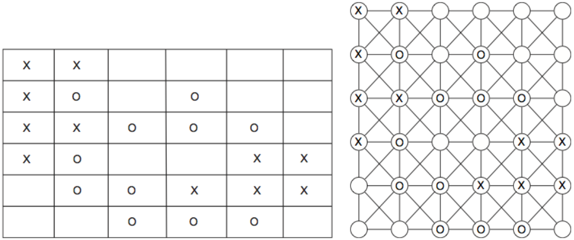
- Freeman's segregation index

$$F = 1 - \frac{e^*}{E(e^*)}$$

$e^* = \frac{e_{AB}}{(e_{AB}+e_{AA}+e_{BB})}$ - observed proportion of between group ties,
$E(e^*) = \frac{2n_A n_B}{(n_A+n_B)(n_A+n_B-1)}$ - expected proportion for random ties

- Assortative mixing

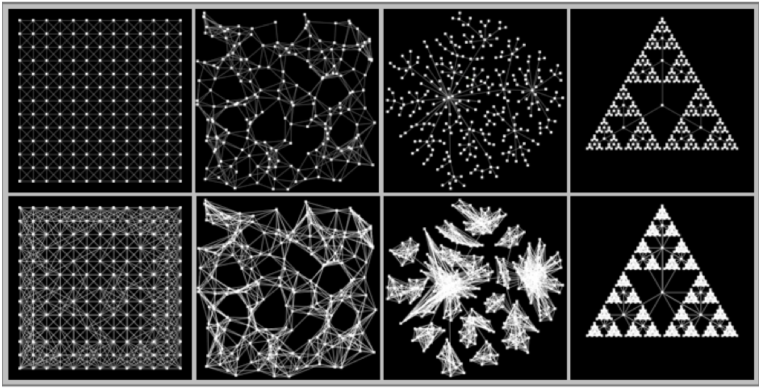
$$Q = \frac{1}{2m} \sum_{ij} (A_{ij} - \frac{k_i k_j}{2m}) \delta(c_i, c_j)$$



(a) (b)

Spatial segregation on networks

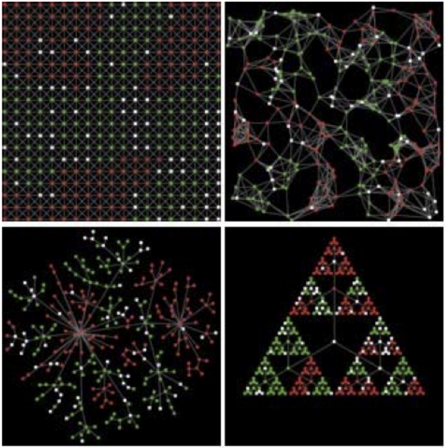
Fixed degree $k = 10$ neighboring graphs: regular, random, scale-free, fractal



Arnaud Banos, 2010

Spatial segregation on networks

$\lambda = 0.5, \theta = 0.8$



Banos, 2010

Summary

- Spatial segregation is taking place even though no individual agent is actively seeking it (minor preferences, high tolerance)
- Network structure does affect segregation
- Fixed characteristics (race) can become correlated with mutable (location)

References

- Contagion, S. Morris, Review of Economic Studies, 67, p 57-78, 2000
- Maximizing the Spread of Influence through a Social Network, D. Kempe, J. Kleinberg, E. Tardos, 2003
- Influential Nodes in a Diffusion Model for Social Networks, D. Kempe, J. Kleinberg, E. Tardos, 2005
- Efficient Influence Maximization in Social Networks, W. Chen, Y. Wang, S, Yang, KDD 2009.
- A Simple Model of Global Cascades on Random Networks. D. Watts, 2002.

References

- Dynamic Models of Segregation, Thomas C. Schelling, 1971
- Segregation in Social Networks, Linton Freeman, 1978
- Gauvin L, Vannimenus J, Nadal JP. Phase diagram of a Schelling segregation model. The European Physical Journal B, 70:293-304, 2009
- Arnaud Banos. Network effects in Schelling's model of segregation: new evidences from agent-based simulations. 2010